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ABSTRACT

Procedures for the development of the Second Assessment of Mathematics (1977-78) by the National Assessment of Educational Progress are outlined following an overview of the National Assessment effort and the first assessment of mathematics during the 1972-73 school year. The framework for organizing the mathematics objectives for the second assessment is given as well as the development of several special topics to be assessed. Appendices contain a list of consultants for objectives, a content outline, and sample exercises. (MN)

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MATHEMATICS OBJECTIVES
Second Assessment

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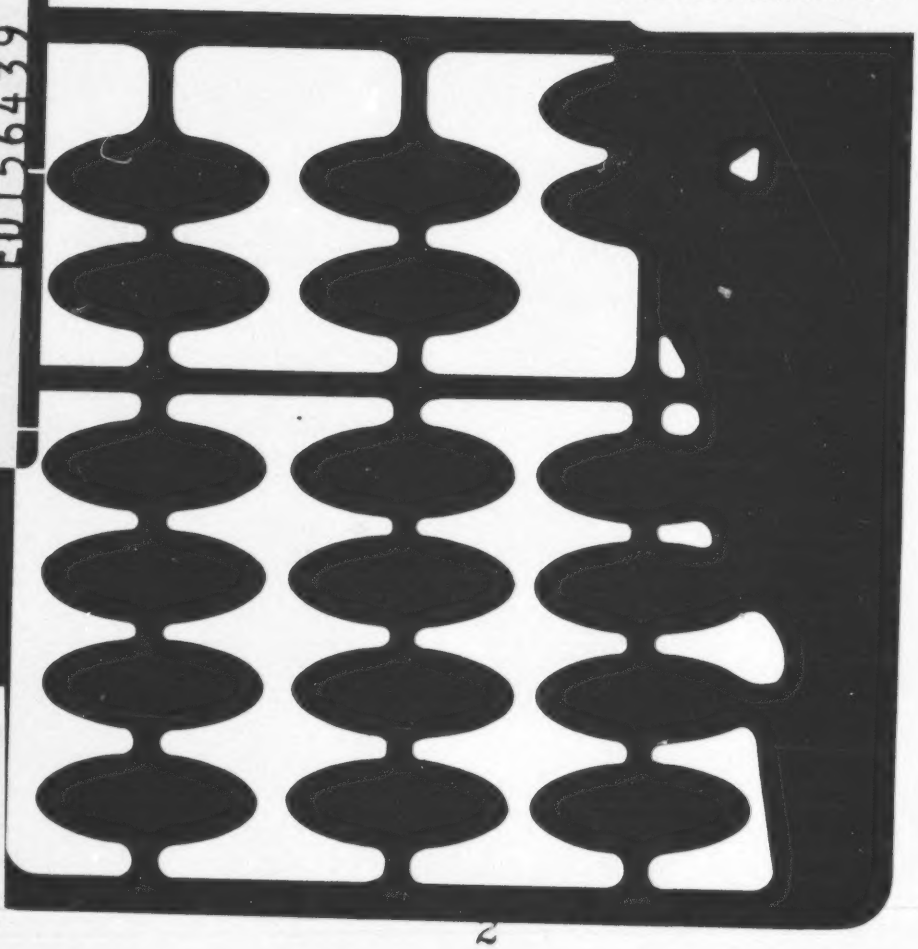

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MATHEMATICS OBJECTIVES
Second Assessment

by the
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CHAPTER 1

OVERVIEW OF NATIONAL ASSESSMENT

History and Purpose

When the U.S. Office of Education was founded in 1867, one charge set before its commissioners was to determine the nation's progress in education. That century-old charge is only now being answered by the National Assessment of Educational Progress (NAEP), a project of the Education Commission of the States, which is funded by the National Center for Education Statistics.

By the early 1960s, the average annual expenditure of public funds for the formal education of young Americans was $30 billion.[1] Yet criticisms of the education system abounded. Defenders of the education establishment found it increasingly difficult to provide evidence that the schools were satisfactorily meeting the education needs of a modern, technological society. The only readily available measures of education quality resulting from this public investment of funds were input measures such as teacher-student ratios and per-pupil expenditures. The tenuous assumption was made that the quality of educational outcomes — what students do or do not know and can or cannot do — was directly related to the quality of the inputs to the education system. There has been no conclusive empirical evidence to support this assumption. The typical standardized achievement tests administered by schools or states provided scores whereby one student could be compared with other students. Such information was useful in categorizing students; however, it provided little information about what students were or were not actually learning. There has been no direct assessment of educational outcomes.

This insufficiency of information became the concern of Dr. Francis Keppel, U.S. commissioner of education (1962–65). He initiated a series of conferences to explore ways to provide the necessary information. In 1964, as a result of these conferences, John W. Gardner, president of the Carnegie Corporation, asked a distinguished group of educators and other concerned persons to form the Exploratory Committee on Assessing the Progress of Education. This committee, chaired by Dr. Ralph W. Tyler, who had been involved since the earliest conferences, was to examine the possibility of conducting an assessment of educational attainments on a national basis.

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[1] Based on U.S. Census Bureau data. Actual amounts expended were as follows: 1969, $24.7 billion; 1962, $29.4 billion; 1964, $35.9 billion.

After much study, the Exploratory Committee on Assessing the Progress of Education decided that it was feasible to inaugurate an assessment project to fill the information gap regarding the quality of educational outcomes by a periodic assessment of the knowledge, skill, understanding and attitudes in 10 learning areas[2] at four age levels (9, 13, 17 and young adult – ages 26–35). This project, named the Committee on Assessing the Progress of Education, began its charge under the auspices of the Carnegie Corporation by assessing the learning areas of science, citizenship and writing in the spring of 1969. Later that same year, the project came under the auspices of the Education Commission of the States. Funding and monitoring were transferred to the National Center for Education Statistics, and the project was renamed the National Assessment of Educational Progress.

Goals of the National Assessment

National Assessment provides information to education decision makers and practitioners that can be used to identify education problem areas, to establish education priorities and to determine the national progress in education. To do so, National Assessment must remain flexible enough to accommodate possible extensions, refinements and modifications. The following goals have been established for the project by the National Assessment Policy Committee, the Analysis Advisory Committee and National Assessment staff.

Goal I: To measure change in the educational attainments of young Americans.

Goal II: To make available on a continuing basis comprehensive data on the educational attainments of young Americans.

Goal III: To utilize the capabilities of National Assessment to conduct special interest "probes" into selected areas of educational attainment.

Goal IV: To provide data, analyses and reports understandable to, interpretable by and responsive to the needs of a variety of audiences.

Goal V: To encourage and facilitate interpretive studies of NAEP data, thereby generating implications useful to education practitioners and decision makers.

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[2] Art, career and occupational development, citizenship, literature, mathematics, music, reading, science, social studies and writing.

Goal VI: To facilitate the use of NAEP technology at state and local levels when appropriate.

Goal VII: To continue to develop, test and refine the technologies necessary for gathering and analyzing NAEP achievement data.

Goal VIII: To conduct an ongoing program of research and operational studies necessary for the resolution of problems and refinement of the NAEP model. (Implicit in this goal is the conduct of research to support previously mentioned goals.)

Methodology

To measure the nation's educational progress, National Assessment estimates the percentage of respondents (at any of four age levels) who are able to acceptably answer a question or perform a task. Each question or task (called an exercise) reflects an education goal or objective. The exercises are administered to scientifically selected samples that take into account such variables as size and type of community, race and geographic region. Students are sampled at three age levels that represent education milestones attained by most students: age 9, when most students have been exposed to the basic program of primary education; age 13, when most students have finished their elementary school education; and age 17, when most students are near completing their secondary education. To accurately reflect the skills, knowledge and attitudes of the 17-year-olds, National Assessment also samples 17-year-olds not enrolled in school. Young adults (ages 26 to 35) are assessed to determine the skills, knowledge and attitudes of those who have completed their formal education and have probably been away from school for a number of years. The samples are designed so that valid inferences can be made about the populations from which the samples were selected.

National Assessment does not use total performance scores for individual respondents because its main concern is how various groups of individuals perform on specific exercises. Thus, it is not necessary for each respondent to take every exercise. The exercises are sorted into booklets, and each in-school respondent takes only one booklet. The various booklets are administered to statistically equivalent samples so that group comparisons can be made across booklets. This technique allows National Assessment to assess performance on far more exercises than would be possible in the usual one-hour testing situation and provides broader coverage of the objectives for each learning area.

Since individuals are not ranked according to their performance on the assessment materials, National Assessment does not emphasize the use of exercises with high discrimination power. The aim of the project is to describe attainment; this is best accomplished if the exercises used cover the entire spectrum of difficulty, from very easy tasks to the most difficult.

While multiple-choice exercises predominate, many open-ended exercises requiring anywhere from a few words to a long essay as an answer are included in each assessment. Exercise writers are instructed to use the exercise format that provides the best and most direct measure of the objectives being assessed. They are encouraged to develop exercises that employ the use of pictures, tapes, films or practical, everyday items as stimuli. In many assessments, individual interviews and observations of the respondents' problem-solving techniques supplement the usual data-collecting procedures. For example, in music, respondents were asked to sing a song or perform on an instrument; in science, respondents were asked to conduct a small experiment; in social studies, respondents were asked to interpret an election ballot.

National Assessment regards positive attitudes toward or opinions about the various learning areas as important educational attainments. Therefore, affective exercises and attitude survey questions are also included in most assessments.

Assessment exercises are administered either to individuals or to small groups (generally less than 25) by specially trained personnel. Exercises are administered to all out-of-school respondents in a one-to-one situation. Some of the in-school respondents are assessed in a one-to-one situation. This proportion varies according to age level, learning area and assessment year. Exercises specifically designed for individual administration include those having unusual stimuli or requiring something other than a written response. In group administrations, instructions and the exercises themselves are presented to the respondents on paced tape recordings to assure complete and uniform presentation of instructions and to give those who have a reading problem a chance to hear the exercise as they are reading it.

To report the nation's educational progress, the project releases up to one-half of the exercises administered in a learning area. Released exercises allow the public to evaluate and use the exercises and the accompanying data. The other exercises are kept confidential and are used to assess performance changes over time.

How do National Assessment test booklets differ from standardized achievement tests? Standardized achievement tests are norm-referenced; NAEP tests are

content- or objective-referenced. On a standardized achievement test, each respondent takes every exercise, receives a score for his performance and is ranked on the basis of that score with respect to a reference group. No respondent takes all of the exercises National Assessment uses to assess a learning area, no respondent receives a score and emphasis is placed on the performance of groups of respondents on specific exercises. Standardized achievement test items are usually limited to a multiple-choice format; National Assessment employs a wide variety of exercise formats. Standardized tests usually focus on the cognitive domain; National Assessment usually includes exercises relating to the affective domain as well. A respondent is required to read the items himself when taking most standardized tests; NAEP exercises are read to respondents by a paced tape or the exercise administrator in an interview situation except during the reading assessment. The items on a standardized achievement test are rarely, if ever, made public; National Assessment releases up to half of the exercises used in an assessment to accompany the data.

The Assessment Cycle

A single cycle of a learning area assessment, from objectives development or redevelopment to completion of the basic technical reporting of the data, requires approximately four years. Development takes one and one-half years; one and one-half years are spent preparing for and performing data collection, and one year is required for preliminary analysis and basic reporting.

During the first nine years, the following learning areas have been assessed:

1969 – science, citizenship, writing
1970-71 – reading, literature
1971-72 – music, social studies
1972-73 – science, mathematics
1973-74 – career and occupational development, writing
1974-75 – art, reading
1975-76 – supplemental mathematics, citizenship/social studies
1976-77 – science, basic life skills
1977-78 – mathematics, consumerism

Future assessments will include the regular learning areas as well as small-scale assessments called "special probes." Special probes are topics of special interest that are assessed once and usually at one age (normally 17-year-olds). All the exercises are released if the area is not planned to be reassessed at a later date. Special probes that have been completed by National Assessment are basic life skills and consumerism.

CHAPTER 2

THE FIRST ASSESSMENT OF MATHEMATICS

The National Assessment of Educational Progress conducted the first assessment of mathematics during the 1972–73 school year. The objectives for this assessment had been developed by two educational testing contractors: Educational Testing Service and Psychological Corporation. Each contractor independently developed a set of objectives, relying on its staff, mathematicians and mathematics educators. The final sets of objectives were reviewed by panels of interested lay citizens to decide which set to use in the assessment. Panel members were evenly divided in their preference for the two sets of objectives. In the absence of a strong preference, the Psychological Corporation was asked to continue the development of objectives. In 1968, the Psychological Corporation completed its revision of the objectives. The revision, together with objectives selected from the Educational Testing Service's version, was compiled into a final statement of objectives for the first assessment of mathematics. A booklet containing the statement was published in 1970.

When the objectives for mathematics were first formulated, they were compared to other statements of objectives that had appeared in the mathematics education literature during the preceding 25 years. The objectives for the first assessment were consistent with objectives appearing in the literature. This outcome was both desired and expected since one criterion for the National Assessment objectives was that they be central to prevailing teaching efforts.

A three-dimensional classification scheme was used to categorize the mathematics objectives for the first assessment. One dimension of the scheme was "Uses of Mathematics," which was divided into three major categories:

1. Social mathematics (the mathematics needed for personal living and effective citizenship in our society).

2. Technical mathematics (the mathematics necessary for various skilled jobs and professions).

3. Academic mathematics (the formally structured mathematics that provide the basis for an understanding of various mathematical processes).

Another dimension of the matrix was "Content." The content areas were:

1. Number and numeration concepts.
2. Properties of numbers and operations.
3. Arithmetic computations.
4. Sets.
5. Estimation and measurement.
6. Exponents and logarithms.
7. Algebraic expressions.
8. Equations and inequalities.
9. Functions.
10. Probability and statistics.
11. Geometry.
12. Trigonometry.
13. Mathematical proof.
14. Logic.
15. Miscellaneous topics.
16. Business and consumer mathematics.
17. Attitude and interest.

The third dimension of the classification scheme consisted of six cognitive "Objectives or Abilities":

1. To recall and/or recognize definitions, facts and symbols.
2. To perform mathematical manipulations.

3. To understand mathematical concepts.

4. To solve mathematical problems – social, technical and academic.

5. To use mathematics and mathematical reasoning to analyze problem situations, define problems, formulate hypotheses, make decisions and verify results.

6. To appreciate and use mathematics.

During the development and review of the exercises, the content and ability dimensions of the classification scheme were the most useful. The exercise developers tended not to use the third dimension, uses of mathematics, when classifying exercises. This third dimension tended to pose too many restrictions on exercise development to make its use worthwhile.

Although the exercises were classified by content and ability, not all content areas or abilities were assessed equally. Certain content topics were purposely measured in more detail than others. Although the objectives were intended to include all the mathematics taught in the nation's schools, it was impossible to measure every objective in depth. Little emphasis, for example, was placed on the topics of trigonometry and logic. The content area of "attitude and interest" and the related ability of "appreciation and use of mathematics" were not measured because the exercises developed to assess these were considered inadequate.

After the exercises had undergone extensive reviews by mathematicians, mathematics educators and concerned lay persons, the exercises to be used in the assessment were selected and arranged in booklets. The 13-year-olds were assessed during October and November of 1972, 9-year-olds during January and February of 1973, the in-school 17-year-olds during March and April of 1973, and the out-of-school 17-year-olds (dropouts and early graduates) and young adults were assessed from January through June of 1973. The majority of the exercises were open-ended (free response) and required the respondent to supply the answer. Respondents were asked to do all written calculations or "scratch work" in the assessment booklets. The responses to the open-ended exercises, including any work the respondents did in the booklets, were tabulated in various scoring categories. These tabulations revealed the percentages of respondents making particular types of errors and thus provided some diagnostic information about common mathematical errors. In order to reduce guessing, respondents were instructed to write the words "I don't know" on the answer line or to fill in the oval beside the "I don't know" choice for multiple-choice exercises if they felt that they did not know the answer to a particular exercise.

Most assessment materials were administered to groups of 8 to 12 respondents at a time, using a paced audio tape to standardize administrations. However, some exercises were administered in a "one-to-one" interview situation in which the exercises were presented to the respondent by a trained exercise administrator. These individually administered exercises were used to elicit responses that would be difficult to observe in a group situation, such as the process a respondent used to solve a problem. Although the out-of-school respondents were not assessed in group settings, the audio tape was used to standardize administrations.

Approximately half of the exercises used in the mathematics assessment were released. These exercises were included in various mathematics reports and made available to groups and individuals for their own uses. The unreleased exercises were used again in the second assessment of mathematics to measure changes in educational attainments. A general survey of the results of the first mathematics assessment is provided in *The First National Assessment of Mathematics: An Overview,* Report 04-MA-00.[1] The text of each released exercise and accompanying documentation including results can be found in the *Mathematics Technical Report: Exercise Volume,* Report 04-MA-20. Data are provided for all of the mathematics exercises, but the exact text and scoring guides are provided for the released exercises only.

Results concerning computational abilities of young Americans are presented and discussed in a special report, *Math Fundamentals: Selected Results From the First National Assessment of Mathematics,* Report 04-MA-01. The results concerning consumer mathematics are presented and discussed in *Consumer Math: Selected Results From the First National Assessment of Mathematics,* Report 04-MA-02. In addition, National Assessment has produced computer data tapes containing respondent-level data for the released mathematics exercises. These data tapes are available to any researcher who wishes to conduct further analysis of the data.[2]

National Assessment has worked closely with a panel of mathematics educators from the National Council of Teachers of Mathematics (NCTM) who studied the

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[1] National Assessment reports can be ordered through the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402.

[2] Data tapes are available, at a charge, through the Data Processing Department, National Assessment of Educational Progress, Suite 700, 1860 Lincoln Street, Denver, Colorado 80295.

data in order to draw implications from the results of the first mathematics assessment. The NCTM panel presented summaries of its findings in the October 1975 issues of *The Arithmetic Teacher* and *The Mathematics Teacher.* Additional brief articles on specific content topics were presented in the October 1975 through May 1976 issues of *The Arithmetic Teacher.* These articles suggest some of the ways mathematics teachers might use information from the first assessment. References for these articles are found in a section at the back of the book entitled "List of References."

CHAPTER 3

PROCEDURES FOR THE DEVELOPMENT OF THE SECOND ASSESSMENT OF MATHEMATICS (1977–78)

Development of Objectives

Unlike the first assessment, which made use of outside contractors, the objectives and exercises for the second assessment were developed through conferences organized by the National Assessment of Educational Progress. The conference procedure was intended to give the assessment greater flexibility, involve more professionals in mathematics and education and be more efficient in cost and time.

Several types of consultants participated in the development conferences: college or university mathematics educators, mathematicians, classroom teachers and interested lay citizens. The objectives had to be acceptable to these groups. The mathematics experts generally had to agree that the objectives were worthwhile and important to assess. The classroom teachers had to consider the objectives to be desirable teaching goals in most schools. Finally, the objectives had to be considered desirable by the lay citizens. The lay group, including parents and others with an interest in education, had to agree that an objective be important for America's youth to achieve and that it be of value in today's society.

Development of the second cycle objectives began with two conferences devoted to reviewing the first-cycle objectives and suggesting new objectives for the 1977–78 assessment. Two groups of mathematics educators, mathematicians and teachers were independently asked to list objectives they thought were important. The result was two outlines of objectives with different formats but similar content. (there was about 80% agreement between the two lists). Three members from each group met during a third conference to synthesize the two outlines.

The objectives were organized into a content by process matrix (see page 17). This matrix resembled the classification scheme developed for the first assessment, but in the second assessment the matrix was used more extensively in the development process. Fewer but more inclusive content and process headings in the matrix for the second assessment resulted in fewer cells. The complicated task of exercise development was subdivided into units corresponding to the cells of the matrix. The cells were weighted in proportion to their

importance. The number of exercises to be assessed in each cell was determined by the relative weights, and review and selection was done by cells.

The objectives matrix was put to the test immediately after the first three objectives conferences. National Assessment held three conferences to write exercises for 9-, 13- and 17-year-olds. Each group of writers wrote exercises for a given age group using the cells of the matrix to organize their work. The NAEP staff and consultants thought that this preliminary set of exercises was acceptable and that the objectives matrix provided a suitable framework for the subsequent development of the assessment.

After the initial objectives matrix was developed, an advisory board was formed to give direction and advice to NAEP staff for the further refinement of the objectives and development of the assessment. The six-member advisory board included three university mathematics educators, two mathematicians and a mathematics teacher. This board was instrumental in organizing the final set of objectives, planning the development of exercises, selecting the final exercises and planning the subsequent report topics.

The first task of the advisory board was to review the new objectives matrix and put it into final form. The board and NAEP staff thought the objectives should be a plan or framework for exercise development and for reporting. They thought the second assessment would be well-organized and more comprehensive than the first assessment if a reporting scheme existed prior to the exercise development. The advisory board devised a set of questions that related to each of four cognitive processes (see pages 22-23). For example, under mathematical skills, one of the questions was "How well can students perform computation?" Under mathematical applications, one of the questions was "How well can students solve typical textbook problems?" Each series of questions was intended to be "answered" by the results of a set of exercises. This planning assured that the questions could be adequately covered by exercises in the assessment.

While the objectives were being formulated, conferences were held on special topics reflecting current trends in mathematics education. The first series of conferences was devoted to developing objectives and sample exercises for computer literacy and computer-related mathematics. Later conferences discussed the assessment of the impact of the hand calculator. These conferences were held to determine how exercises using the hand calculator could be incorporated into the assessment.

A special topic that received considerable attention was the measurement of attitudes toward mathematics. Attitudes were not measured during the first

mathematics assessment because of the difficulty of developing adequate exercises. However, consultants for the second assessment were virtually unanimous in encouraging the development of attitude exercises; and a concentrated effort was made to develop such measures.

Development of Exercises

The development of exercises began after the objectives matrix was completed. The first writing groups used the cognitive-process questions as a guide for the development of exercises. A writing group would select one or two questions under one of the four major process headings and develop a set of exercises that they thought would help answer a question. Each writer's exercises were reviewed by the entire group. Reviews addressed age-level appropriateness, accuracy of the content, how well the exercise measured the objective and readability. This process usually took approximately six weeks. Exercises passing the group review were edited by the National Assessment staff to fit the NAEP format and technical requirements.

The exercises produced by the writing groups were tried out in the schools to collect item statistics, timing information and scoring information, and to discover problems in wording, directions, scoring or administration procedures. The tryout schools were selected to represent high- and low-income communities as well as more typical communities. The tryouts were administered to students in at least four classrooms (approximately 100 students) at the desired ages. So that the tryouts would closely simulate the final assessment field procedures, the students recorded their answers in the test booklets; directions and questions were read to them on an audio tape; and National Assessment staff members, rather than classroom teachers, administered the tests. The completed tryout packages were scored and tabulated. The item-analysis statistics, as well as the administrator's reports of any field problems, helped both staff and consultants to evaluate and revise the exercises. Revised exercises were then tried out again.

After the initial exercise pool was developed by writing groups working within the framework of the process dimension of the matrix, three writing teams reviewed the exercises and accompanying tryout data organized by content categories. The set of exercises in a category was reviewed for content coverage. The team then edited the existing exercises and wrote new exercises as needed. These exercises were field tested, then final selection was made. These teams frequently brought in their own consultants for specific expertise.

Each exercise in the final pool was reviewed in a series of conferences by at least 25 different consultants: mathematicians, mathematics educators, classroom

teachers and interested lay citizens. Exercises for each age group were reviewed for appropriateness by teachers who taught students of that age. A lay panel representing a variety of occupations and interests also reviewed the exercises. The lay reviewers looked for sex or racial bias in the exercises and at the general importance of each exercise.

The final exercise selection was approved by the advisory board. The number of exercises that could be assessed was limited by the amount of testing time available at each age: 245 minutes at age 9 (7 booklets), 385 minutes at age 13 (11 booklets) and 420 minutes at age 17 (12 booklets). There was not enough testing time to include all of the exercises in the final exercise pool. The advisory board used the assigned cell weights in making judgments on number of exercises to include in the assessment for each objective.

An attempt was made to make the test booklets parallel with respect to the objectives measured and with respect to difficulty. The exercises measuring an objective were scattered throughout the booklets so that as many students as possible could respond to them. Recycled exercises (those exercises from the first assessment being reassessed) were packaged together if they had been together in the first assessment. In addition, all of the unreleased exercises appearing in a package at a particular age in the first assessment were placed in the same order relative to each other and appeared together either at the beginning or end of the booklets.

Several packages at each age required the use of apparatus such as compasses and English or metric rulers, which were supplied to the students. One test booklet at each age required the use of a simple four-function hand calculator, which was also supplied. Some of the exercises in the calculator booklets also appeared in booklets for which the calculator was not used.

Individual interview exercises were not used in the second assessment. Such exercises in the first assessment were designed to examine the processes students use to solve problems. National Assessment considered this goal important, but given limited resources, the advisory board and NAEP staff reluctantly decided to measure a greater number of exercises in group settings. The expense of administering an interview booklet is much greater than that of a group booklet. Also, data about a student's problem-solving processes — however vital to teachers and researchers — are difficult to summarize and to relate to questions regarding levels of educational attainment.

The field-assessment procedure was similar to that described in Chapter 2 for the first assessment. The packages were administered to a scientifically selected sample of students. Thirteen-year-olds were assessed in the fall, 9-year-olds in the winter and 17-year-olds in the spring.

CHAPTER 4

FRAMEWORK FOR THE OBJECTIVES

The National Assessment staff, advisors and consultants concluded that a two-dimensional matrix was the most efficient framework for organizing the mathematics objectives for the second assessment. One dimension includes four cognitive processes, and the other dimension includes five mathematics content areas:

		CONTENT				
		A. Numbers and Numeration	B. Variables and Relationships	C. Shape, Size and Position	D. Measurement	E. Other Topics
PROCESS	I. Mathematical knowledge					
	II. Mathematical skill					
	III. Mathematical understanding					
	IV. Mathematical application					

Content

The content domain for the second assessment of mathematics draws primarily from the current curriculum of elementary and secondary schools, although some projection of future mathematics emphases is acknowledged (for example, greater use of metric measures, earlier introduction of decimals and calculator computation). Mathematics up to, but not including, the calculus is included in the assessment exercises, which are classified according to content categories. There are five content categories used in this assessment:

A. Numbers and numeration.
B. Variables and relationships.
C. Size, shape and position.
D. Measurement.
E. Other topics.

These content categories helped to organize the domain, but were not intended to be represented equally in the assessment. The selection of exercises was based on judgments about the appropriateness for each age group and the relative emphasis in the school curriculum.

Numbers and Numeration

This category contains the largest number of exercises because of its importance in the curriculum. Exercises deal with the ways numbers are used, processed or written. Knowledge and understanding of numeration and number concepts are assessed for whole numbers, fractions, decimals, integers and percents, with considerable emphasis placed on operations. Number properties and order relations are also included. Problem-solving exercises include routine number problems, nonroutine problems and consumer problems. Nonroutine problems are exercises not normally encountered in the curriculum, but understandable to the age group. Consumer problems deal primarily with the uses of mathematics in commercial situations (for example, buying and selling, interpreting graphs and saving money) and are emphasized more at the 17-year-old level than at the two younger age levels.

Variables and Relationships

The use of variables and relationships corresponds to an important part of the school mathematics curriculum. The exercises for this content category deal with facts, definitions and symbols of algebra; the use of variables in equations and inequalities; the use of variables to represent elements of a number system; functions and formulas; coordinate systems; and exponential and trigonometric functions. There are very few exercises appropriate for 9-year-olds in this category, and only a few topics are appropriate for 13-year-olds. Many more are appropriate at the 17-year-old level.

Shape, Size and Position

The exercises in this content category measure objectives related to school geometry. The emphasis in the assessment is not on geometry as a formal deductive system. The exercises concern plane and solid shapes, congruence, similarity, properties of triangles, properties of quadrilaterals, constructions, sections of solids, other basic theorems and relationships, rotations and symmetry.

Measurement

A large portion of the assessment is devoted to measurement, reflecting increased emphasis on measurement in the school curriculum. The exercises

cover appropriate units; equivalence relations; instrument reading; length, weight, capacity, time and temperature; perimeter, area and volume; non-standard units; and precision and interpolation. A substantial number of the measurement exercises require the use of metric units.

Other Topics

Other mathematical content topics included in this assessment at all age levels are probability and statistics; graphs, tables and charts; and logic. Special assessment exercises and procedures have been developed to assess attitudes related to mathematics, computer literacy and the use of the hand calculator.

Appendix B is a summary outline of the content categories described above. The distribution of exercises by content category and age group is shown in Figure 1.

FIGURE 1. Approximate Number of Exercises by Age and Content

CONTENT	Age 9	Age 13	Age 17
A. Numbers and numeration	110	150	160
B. Variables and relationships	20	40	90
C. Shape, size and position	30	50	70
D. Measurement	40	50	60
E. Other topics	30	60	50

Process

The process domain for the second assessment comprises four categories:

I. Mathematical knowledge.
II. Mathematical skill.
III. Mathematical understanding.
IV. Mathematical application.

Like the content domain, the process domain can be used to classify either objectives of mathematics instruction or exercises to assess the learning of mathematics. Although each category suggests a type of mental process, neither objectives nor exercises fall neatly into a single process category – if only because the process has to be inferred; and different people may use different processes or different combinations of processes. Arbitrary decisions must be made in using any system of process categories. Such a system is helpful, however, in ensuring consideration of the diversity possible within a given content category.

Mathematical Knowledge

Mathematical knowledge refers to the recall and recognition of mathematical ideas expressed in words, symbols or figures. Mathematical knowledge relies, for the most part, on memory processes. It does not ordinarily require other, more complex mental processes.

Exercises that assess mathematical knowledge require that a person recall or recognize one or more items of information. An example of an exercise involving recall would be one that asks for a multiplication fact such as the product of five and two. Another example would be an exercise asking for the statement of a mathematical relationship such as the law of cosines. An example of an exercise involving recognition would be one that presents several symbols and asks which symbol means "parallel."

Mathematical Skill

Mathematical skill refers to the routine manipulation of mathematical ideas. Mathematical skill relies on algorithmic processes. An algorithm is a standard procedure that always leads to an answer. Mathematical skill requires the recollection of how to use the algorithm.

Exercises that assess mathematical skill assume that the required algorithm has been learned and practiced. They do not require that the person decide which algorithm to use or that he or she apply the algorithm to a new situation. Such an exercise aims at measuring proficiency in carrying out the algorithm rather than the understanding of how or why it works. Mathematical skill is assessed by exercises that require the performance of specified tasks such as making measurements, multiplying two fractions, graphing a linear equation or bisecting an angle.

Mathematical Understanding

Mathematical understanding refers to the explanation and interpretation of mathematical knowledge. Mathematical understanding relies primarily on translation processes. The mathematical knowledge can be expressed in words, symbols or figures; and the translation may be within or between any of these modes of expression. Mathematical understanding involves memory processes as well as processes of associating one item of knowledge with another.

Exercises that assess mathematical understanding require that a person provide an explanation or an illustration for one or more items of knowledge. They require the transformation of knowledge but not the application of that knowledge to the solution of a problem. An example of an exercise involving explanation would be one that asks why a certain graph is not the graph of a function. Examples of exercises involving interpretation would be ones that ask for a drawing of an array to represent six times seven or that ask for an equation to represent the information in a word problem.

Mathematical Application

Mathematical application refers to the use of mathematical knowledge, skill and understanding. Mathematical application relies on memory, algorithmic, translation and judgment processes.

Exercises that assess mathematical application require a sequence of processes that relate to the formulation, solution and interpretation of problems. The processes may include recalling and recoding knowledge, selecting and carrying out algorithms, making and testing conjectures and evaluating arguments and results. An exercise might require the solution of a standard problem on proportion, or it might require the demonstration that two geometric figures are congruent. It might require the estimation of the amount of carpet needed for a room or the formulation of a problem given a graph of statistical data. Exercises assessing mathematical application may vary from routine textbook problems to exercises dealing with mathematical arguments. In these exercises the person is not told exactly what to do or how to do it. He or she must use reasoning and decision-making processes as well as mathematical knowledge, skill and understanding.

The distribution of exercises by age group and process category is shown in Figure 2.

FIGURE 2. Approximate Number of Exercises by Age and Process

	Age 9	Age 13	Age 17
Mathematical knowledge	45	45	55
Mathematical skill	65	85	110
Mathematical understanding	60	105	105
Mathematical application	60	115	160

Questions To Be Answered by the Assessment

In the development and selection of exercises for the assessment, care was taken to assure an appropriate balance of emphasis on both the content and process dimensions. Achieving this balance was facilitated by the use of a set of questions for which the assessment would provide some answers. These questions are organized according to the categories of the process dimension. In general, the items related to each of these questions reflects topics from several or all of the content categories.

The questions are based on the combined priorities of the interested public, mathematicians, mathematics educators, education administrators and teachers. The assessment is aimed at answering questions related to students' knowledge of mathematical facts, their computational performance, their mathematical understanding and their performance in solving problems. The questions are grouped into four process categories, described previously, and a separate category reflecting topics of special interest, which are discussed in Chapter 5.

I. Mathematical knowledge

   A. How well can students recall and recognize facts, definitions and symbols?

II. Mathematical skill

   A. How well can students perform computations, including computations with whole numbers, integers, fractions, decimals, percents, ratios and proportions?

B. How well can students make measurements?

C. How well can students read graphs and tables?

D. How well can students perform geometric manipulations like constructions and spatial visualizations?

E. How well can students perform algebraic manipulations?

F. How well can students estimate the answers to computations and measurements?

III. Mathematical understanding

A. How well can students translate a verbal statement into symbols or a figure, and vice versa?

B. How well do students understand mathematical concepts and principles?

IV. Mathematical application

A. How well can students solve typical textbook problems?

B. How well can students solve nonroutine problems?

C. How well can students estimate the answers to problems?

D. How well can students use mathematics in reasoning and making judgments?

The development and selection of exercises was primarily organized around these questions rather than the broadest possible range of content and process objectives. These questions, which express the main objectives of the second mathematics assessment, serve to organize the intended reports.

To answer these questions as comprehensively as possible, sets of related or "nested" exercises appear in the assessment. For example, the same numbers may be used in a computation exercise and an application exercise; or identical data may be provided in several different formats. Nested exercises are an attempt to identify the mathematical processes that cause students difficulty.

Sample Exercises

Two sample exercises for each of the process-dimension questions are included in Appendix C. They have been selected to show the type of exercises included under each question, the range of difficulty and the variety of formats found in the assessment.

CHAPTER 5

DEVELOPMENT AND ASSESSMENT OF SPECIAL TOPICS

During the development of objectives, several special topics to be assessed were identified. These topics either did not fit neatly into the objectives matrix, or they were considered important enough to require special emphasis in the assessment. In some cases, these topics had not been assessed before; and special procedures and exercise formats had to be developed.

The special topics included in the second assessment were the following: attitudes toward mathematics, the hand calculator, computer literacy, consumer mathematics and metrication. These topics are discussed in this chapter.

Attitudes Toward Mathematics

The development process began with a review of the literature on attitudes and the collection of information and exercises from several existing attitude scales. Some exercises were modified, and new exercises were written to measure important attitudes. The exercises were analyzed and sorted into four major categories: (1) Mathematics in School, (2) Mathematics and Oneself, (3) Mathematics and Society and (4) Mathematics as a Discipline. The major difference between the attitude measures developed for the second assessment and previously existing measures is that the National Assessment exercises are not aggregated into scales. The exercises within each category are, however, related by content and permit an analysis of the attitudes of various groups of students.

Field testing revealed that the "Mathematics as a Discipline" category was not appropriate for 9-year-olds. Furthermore, exercises for 9-year-olds require fewer and simpler response options than exercises for the two older age groups. The attitude exercises are identical for 13- and 17-year-olds.

The first category, "Mathematics in School," consists of exercises designed to assess attitudes toward the mathematics encountered in school. Subcategories of exercises include a school subject comparison, a breakdown of classroom activities on the basis of frequency of occurrence and students' attitudes toward those activities, and a similar breakdown of mathematics content activities.

The subject-comparison exercises list the most commonly encountered school subjects (e.g., science, mathematics, social studies); and respondents are asked to

rate each subject on the basis of whether they like or dislike it, whether they find it easy or hard and whether or not they view it as important. Older respondents are asked to indicate whether or not they have studied a particular subject.

Students are presented with a list of activities that might occur in a mathematics classroom and are asked to indicate how often (often, sometimes, never) they have participated in such activities. They are also asked to state whether they like or dislike each activity and whether they find the activity useful in helping them learn mathematics. Typical exercises are the following:

Last year in your mathematics course how often did you

A. watch the teacher work mathematics problems on the board?

Often ⬭ Sometimes ⬭ Never ⬭

B. get individual help from the teacher on your mathematics?

Often ⬭ Sometimes ⬭ Never ⬭

C. help a classmate do mathematics?

Often ⬭ Sometimes ⬭ Never ⬭

How do you feel about each of these activities in learning mathematics? First, how much do you like or dislike them? Second, how useful are they in learning mathematics? Indicate your feeling by filling in one oval on each line.

A. Taking mathematics tests

Like A Lot ⬭ Like ⬭ Undecided ⬭ Dislike ⬭ Dislike A Lot ⬭

Very Useful ⬭ Useful ⬭ Undecided ⬭ Not Very Useful ⬭ Useless ⬭

B. Doing mathematics homework

Like A Lot	Like	Undecided	Dislike	Dislike A Lot
⬭	⬭	⬭	⬭	⬭

Very Useful	Useful	Undecided	Not Very Useful	Useless
⬭	⬭	⬭	⬭	⬭

Respondents are also given a list of content activities and asked to respond on an important/not-important, easy/hard, like/dislike basis. Sample exercises include:

How do you feel about each of these mathematics activities? First, how important are they? Second, how easy are they? Third, how much do you like them? Indicate your feeling by filling in one oval on each line.

A. Solving word problems

Very Important	Important	Undecided	Not Very Important	Not Important At All
⬭	⬭	⬭	⬭	⬭

Very Easy	Easy	Undecided	Hard	Very Hard
⬭	⬭	⬭	⬭	⬭

Like It A Lot	Like It	Undecided	Dislike It	Dislike It A Lot
⬭	⬭	⬭	⬭	⬭

B. Working with fractions

Very Important	Important	Undecided	Not Very Important	Not Important At All
⬭	⬭	⬭	⬭	⬭

Very Easy	Easy	Undecided	Hard	Very Hard
⬭	⬭	⬭	⬭	⬭

Like It A Lot	Like It	Undecided	Dislike It	Dislike It A Lot
⬭	⬭	⬭	⬭	⬭

C. Estimating answers to problems

Very Important	Important	Undecided	Not Very Important	Not Important At All
⬭	⬭	⬭	⬭	⬭
Very Easy	Easy	Undecided	Hard	Very Hard
⬭	⬭	⬭	⬭	⬭
Like It A Lot	Like It	Undecided	Dislike It	Dislike It A Lot
⬭	⬭	⬭	⬭	⬭

Exercises within the "Mathematics and Oneself" category assess a respondent's perceptions of himself or herself in relationship to mathematics. Different attitude components such as anxiety, motivation, self-concept and enjoyment of mathematics are reflected in these Likert-type exercises. Sample exercises are the following:

This exercise asks how you feel about mathematics or mathematics activities. There are no correct answers. The answer choices are "True about me," "Sometimes true about me" and "Not true about me." For each part, choose the one response that best describes how you feel about the statement. Be sure to fill in one oval in each box.

A. Mathematics is boring for me.

True About Me	Sometimes True About Me	Not True About Me
⬭	⬭	⬭

B. I usually understand what we are talking about in mathematics.

True About Me	Sometimes True About Me	Not True About Me
⬭	⬭	⬭

C. Doing mathematics makes me nervous.

True About Me	Sometimes True About Me	Not True About Me
⬭	⬭	⬭

The three examples above are for 9-year-olds. For 13- and 17-year-olds, the exercises have five response options, from "Strongly Disagree" to "Strongly Agree."

The "Mathematics and Society" category reflects two major concerns: the value of mathematics to the individual as a member of society and the value of mathematics to society in general. Likert-type exercises are designed to assess attitudes toward both the usefulness and importance of mathematics to society. Typical exercises are the following:

This exercise asks how you feel about mathematics or mathematics activities. There are no correct answers. The answer choices are "Strongly Disagree," "Disagree," "Undecided," "Agree" or "Strongly Agree." For each part, choose the one response that best describes how you feel about the statement. Be sure to fill in one oval in each box.

A.	Most people do not use mathematics in their jobs.				
	Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
	○	○	○	○	○
B.	I would like to work at a job that lets me use mathematics.				
	Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
	○	○	○	○	○

Attitudes toward mathematics as a field of study are the focus of the exercises in the "Mathematics as a Discipline" category. Students' views toward mathematics as a cumulative or compartmentalized subject, the status of mathematics as a fixed or changing subject and mathematics as a process are among the different aspects of attitude assessed in this category. Items such as the following are included:

This exercise asks how you feel about mathematics or mathematics activities. There are no correct answers. The answer choices are "Strongly Disagree," "Disagree," "Undecided," "Agree" or "Strongly Agree." For each part, choose the one response that best describes how you feel about the statement. Be sure to fill in one oval in each box.

A.	Mathematics is made up of unrelated topics.				
	Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
	○	○	○	○	○

B.	Mathematics helps one to think logically.				
	Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
	○	○	○	○	○
C.	Doing mathematics requires lots of practice in following rules.				
	Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
	○	○	○	○	○

Hand Calculator

The increasing availability and popularity of hand calculators makes it important for National Assessment to collect information on their use by students. The small, inexpensive calculator has the potential to dramatically influence the mathematics curriculum in the United States.

Proponents of the hand calculator suggest that calculator use will facilitate the learning of mathematics. They believe that students will develop better number sense and estimation skills, learn decimals at a younger age, be able to solve more difficult and realistic application problems and more effectively learn mathematical concepts. Fears related to the use of hand calculators in schools reflect concerns about acquiring basic skills in mathematics. Some people opposed to using calculators in schools are afraid that students will not develop basic computational skills and may become too dependent on calculators.

Several conferences were organized by National Assessment to discuss the place of the hand calculator in the second mathematics assessment. The participants at these conferences identified five categories of exercises for which calculators might be used. These categories were:

1. Routine Computation – This category includes typical computations with whole numbers, decimals, fractions and integers that are routinely taught at a particular age.

2. More Difficult Computations – Students might be asked to perform difficult computations or computations for which algorithms have not been formally taught. For example, 9-year-olds might be asked to do computations with decimals or difficult division problems. Thirteen-year-olds might be given chaining operations or conversions between fractions

and decimals. Students at all ages might be required to work with very large numbers or complicated decimals that would make computation without a calculator tedious.

3. Understanding Concepts – Nine-year-olds might use the calculator to learn more about place value, and 13-year-olds might learn estimation for order of magnitude. Seventeen-year-olds might use the calculator to facilitate learning order of operations, approximating square roots, graphing functions and understanding properties of functions.

4. Exploration – Exercises in this category might deal with topics in number theory including series, summations, patterns or divisibility problems.

5. Applications and Problem Solving – This category includes routine word problems as well as more difficult ones and multistep problems. For 9-year-olds, some problems might use larger numbers than they are accustomed to working with. Thirteen-year-olds might work problems involving percent, unit pricing and other more difficult word problems. Seventeen-year-olds might be given a variety of realistic consumer problems as well as exercises dealing with mathematical formulas.

In an effort to collect data on the use of the hand calculator, as wide a variety of topics as possible was assessed. Although it was agreed that the five categories should be assessed at each of the three age groups, practical considerations imposed limitations on what could be assessed. Some activities using the hand calculator (especially understanding and exploration exercises) are more appropriate for instructional use in the classroom than for an assessment. Thus, of the five categories of exercises, categories one, two and five were measured in the greatest depth.

One test booklet of approximately 25 exercises for calculators was administered at each age. Students are required to use a simple four-function calculator after being given a minimal amount of instruction on how the calculator is used. Many of the exercises in each calculator test booklet appear again in another booklet for the same age group. This allows for comparisons of performance with and without the calculator. Other exercises appeared only in the calculator booklet. These exercises are difficult or too time-consuming without a calculator.

Each student answers background questions regarding his or her experience with calculators. These background questions ask how often the student has used a calculator, if the student's family owns one, in what school classes the student has used a calculator and what experiences with calculators outside of school the student has had.

Computer Literacy

During the initial objectives conferences, the participants expressed interest in and enthusiasm for assessing students' knowledge of computers. Work was done on both computer literacy and what was termed "computer-related mathematics" during the development of the assessment. Because not all students today have access to computers, this assessment collected only limited data on students' knowledge of computers and their uses.

Thirteen- and 17-year-olds are asked to provide background information on their experience with computers: whether they have access to one in their school, what programming language they have learned and what specific activities and problems they have solved using a computer. A series of attitude questions is included that contains computer literacy. Students' views are assessed regarding such topics as the dehumanizing aspects of computers and the effects of computers on privacy and on jobs. Finally, students are asked to identify how useful a computer might be in doing specific tasks or jobs, like printing credit card bills or keeping track of an inventory. A few cognitive exercises ask students to follow a simple mathematical flowchart or BASIC program and indicate the output.

Consumer Mathematics

Consumer education courses in the schools are becoming an important aspect of a young American's education. As incomes increase and the marketplace offers a greater variety of goods and services, then more consumer-related decisions have to be made. Consumer education attempts to teach students how to evaluate products and services, get the best value for their money, find and use reliable information and develop competence in resolving their problems in the marketplace.

The assessment of consumer mathematics covers a very small part of what is typically considered consumer education. Exercises deal with the quantitative aspects of consumer education, including unit pricing, finance charges, savings on sale items, making change, sales tax, commissions, simple interest, installment buying and interpreting graphs. Not only was it difficult for consultants to define exactly what topics should be included in the consumer mathematics portion of the assessment, but limited resources prevented thorough coverage of consumer mathematics. Many of the consumer exercises were specifically developed to answer the question "How well can students apply mathematics to consumer problems?" But other exercises developed for other questions in the

"Mathematical Application" process category also cover consumer topics and may therefore be included under more than one question for reporting purposes.

In addition to the consumer mathematics exercises appearing in the mathematics assessment, a "special probe" assessment covering nonmathematical consumer topics was conducted with 17-year-olds and young adults in 1977–78. Four major objectives were covered in the consumerism special probe: consumer protection, finance, behavior and economics. By combining the results of the consumer mathematics assessment and the special probe, National Assessment should obtain a good picture of consumer skills.

Metrication

In 1974 the U.S. Congress concluded that the metric system would "become the dominant system of weights and measures in the United States . . . . It is the policy of the United States to encourage education agencies and institutions to prepare students to use the metric system of measurement with ease and facility as part of their education program."[1]

Numerous education materials have been developed for use in the schools to teach the metric system of measurement. In response to metrication, the second mathematics assessment included many exercises requiring the use and knowledge of the metric system. Exercises require selecting appropriate units, approximating lengths or weights using metric units, knowing metric prefixes, measuring using metric units and solving word problems with metric units. Conversions between English and metric units are not included in the assessment exercises. A background question asks students what and how much experience they have had with the metric system.

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[1] (20 U.S.C. 1862) Enacted August 21, 1974, Public Law 93-380, sec. 403, 88 Stat. 546.547.

APPENDIX A

CONSULTANTS FOR OBJECTIVES

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APPENDIX B

CONTENT OUTLINE

A. Numbers and numeration

1. Numeration
2. Number concepts
3. Operations with whole numbers
4. Operations with fractions
5. Operations with decimals
6. Operations with integers
7. Percent
8. Relations
9. Properties

B. Variables and relationships

1. Facts, definitions and symbols
2. Use of variables in equations and inequalities
3. Use of variables to represent elements of a number system
4. Functions and formulas
5. Coordinate systems
6. Exponential and trigonometric functions

C. Shape, size and position

1. Plane and solid shapes
2. Congruence
3. Similarity
4. Properties of triangles
5. Properties of quadrilaterals
6. Constructions
7. Sections of solids
8. Other basic theorems and relationships
9. Rotations and symmetry

D. Measurement

1. Appropriate units
2. Unit equivalents
3. Instrument reading

4. Linear measure
5. Area, perimeter, volume
6. Nonstandard units
7. Precision and interpolation

E. Other topics

1. Probability
2. Statistics
3. Graphs, tables, charts
4. Logic
5. Attitudes toward mathematics
6. Computer literacy
7. Hand calculator

APPENDIX C

SAMPLE EXERCISES

IA. How well can students recall and recognize facts, definitions and symbols?

Which is longer?

- ◯ 1 yard
- ◯ 1 meter
- ◯ I don't know.

Ages 9, 13, 17

What is the distance around a circle called?

- ◯ Circumference
- ◯ Diameter
- ◯ Area
- ◯ Radius
- ◯ I don't know.

Ages 9, 13, 17

Which one of these means "less than"?

- ◯ >
- ◯ <
- ◯ =
- ◯ $
- ◯ I don't know.

Ages 9, 13, 17

IIA. How well can students perform computations? (whole numbers, integers, fractions, decimals, percent, ration and proportion)

Find the sums.

A. $\begin{array}{r} 32 \\ +\ 74 \\ \hline \end{array}$

Answer _____

B. $\begin{array}{r} 57 \\ +\ 18 \\ \hline \end{array}$

Answer _____

C. $\begin{array}{r} 328 \\ 411 \\ +\ 293 \\ \hline \end{array}$

Answer _____

Ages 9, 13, 17

A. What number is 125% of 20?

Answer _____

B. 3 is what percent of 60?

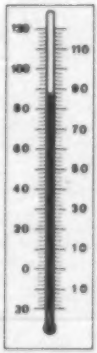
Answer _____

C. 140 is 35% of what number?

Answer _____

Ages 13, 17

IIB. How well can students make measurements?



What temperature is shown on the above thermometer?

○ 84°

42

○ 87°

○ 88°

○ 89°

○ 90°

○ I don't know.

Ages 9, 13, 17

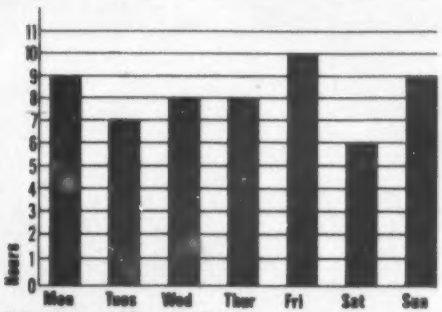
Use your ruler. Measure this line segment to the nearest centimeter.

Answer _____

Ages 9, 13

IIC. How well can students read graphs and tables?

The chart below shows the hours that Jane slept each night of one week.



How long did she sleep on Thursday?

○ 7 hours

○ 8 hours

○ 9 hours

○ I don't know.

Ages 9, 13

Wonderman T-Shirts Sold at
Michaelson's Store During April

	Small	Medium	Large	X-Large
White	9	8	13	11
Red	15	9	10	12
Blue	14	12	8	7

How many large, white Wondermen T-shirts were sold at Michaelson's Store during April?

Answer _____

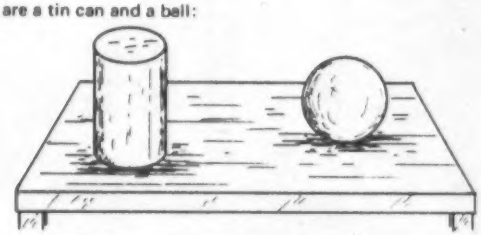
Age 9

IID. How well can students perform geometric manipulations? (constructions and spatial visualizations)

Use your compass and straightedge to bisect this line segment.

Ages 13, 17

Here are a tin can and a ball:



Here is how each looks from the side.

Draw how each looks when you are directly above them.

Ages 9, 13, 17

IIE. How well can students perform algebraic manipulations?

$$\frac{4}{3}n = 24$$

Solve this equation for n.

Answer _____

Ages 13, 17

If $x = 7$, $y = 3$, and $a = 5$, what is the value of $\frac{6x + ay}{3}$

Answer _____

Age 17

IIF. How well can students estimate the answers to computations?

$$\begin{array}{r} 492 \\ -110 \\ \hline \end{array}$$

The answer to this subtraction problem is approximately

- ◯ 200
- ◯ 300
- ◯ 400
- ◯ 500
- ◯ I don't know.

Ages 9, 13

For each problem below, which answer is most reasonable? Estimate the answers in your head.

A. $34 \div .6$

- ◯ 6
- ◯ 20
- ◯ 55

○ 75

○ I don't know.

B. 2.4 x 6.09

○ 9

○ 12

○ 15

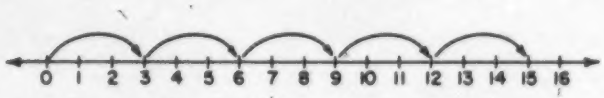
○ 18

○ I don't know.

Ages 13, 17

IIIA. How well can students translate a verbal statement into symbols or a figure, and vice versa?

What multiplication problem is illustrated by this number line picture?



Answer _____ x _____ = _____

Age 9

Matthew has 1800 puzzle pieces. Each puzzle he owns has x pieces. Which number sentence tells how many puzzles (y) Matthew owns?

○ $y = 1800 \div x$

○ $y = 1800 + x$

○ $x \div y = 1800$

○ $y = 1800 \times x$

○ I don't know.

Ages 13, 17

IIIB. How well do students understand mathematical concepts and principles?

What does 4 x 6 mean?

- ◯ 6 + 6 + 6 + 6
- ◯ 1 + 2 + 3 + 4 + 5 + 6
- ◯ 6 x 6 x 6 x 6
- ◯ 4 x 4 x 4 x 4
- ◯ I don't know.

Ages 9, 13

A string is 5 pencils long. The same string is 8 straws long. Which is TRUE?

- ◯ The pencil is longer than the straw.
- ◯ The pencil is shorter than the straw.
- ◯ The pencil and the straw are the same length.
- ◯ I don't know.

Ages 9, 13

IVA. How well can students solve typical textbook problems? (one or multiple step)

One lap around a track is $\frac{1}{4}$ mile. In a race of 2 miles, how many laps are run?

- ◯ 2
- ◯ 4
- ◯ 8
- ◯ I don't know.

Age 13

Debbie made an 8 minute telephone call to a friend in a nearby city. The cost of the call was 34¢ for the first minute and 33¢ for each additional minute. What was the cost of the call?

○ $ .67

○ $2.64

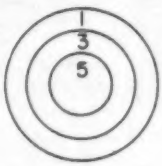
○ $2.65

○ $2.72

○ I don't know.

Ages 9, 13

IVB. How well can students solve non-routine (not typical textbook) problems?

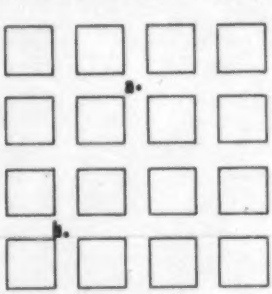


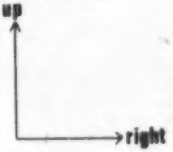
If three darts hit the target, could you get a total score of

		Yes	No	I don't know.
A.	5	○	○	○
B.	9	○	○	○
C.	14	○	○	○

Ages 9, 13

If you want to walk from point b to point a by always going up or to the right, how many different ways can you make the trip? Do not walk across the "blocks."





Answer _____

Age 9

IVC. How well can students estimate the answer to a problem?

John has 10 record albums. Bill has 15 record albums. Tom has 25 record albums. The boys have to carry the albums a long way, so they decided to share the albums so that each boy carries about the same. About how many albums will each boy have to carry?

- ○ 10
- ○ 17
- ○ 25
- ○ 50
- ○ I don't know.

Ages 9, 13

At sea level, water boils at 100° Celsius. For every 900 feet above sea level, the boiling point is lowered by 1° Celsius. At what approximate temperature would water boil in Denver, which is 5280 feet above sea level? Estimate the answer in your head.

- ○ 90° C
- ○ 95° C
- ○ 98° C
- ○ 102° C
- ○ 105° C
- ○ I don't know.

Age 17

IVD. How well can students use mathematics in reasoning and making judgments?

Don is taller than Vicky. Vicky is taller than Marguerite. Which of these statements must be true?

- Marguerite is taller than Vicky.
- Marguerite is taller than Don.
- Don is taller than Sam.
- Don is taller than Marguerite.
- Can't tell.
- I don't know.

Ages 9, 13

A vacumn cleaner distributor employs three part-time salesmen. The table below shows the number of days worked and the total amount of sales for each of the salesmen for the past nine months.

Salesman	Days Worked	Total Sales
Wilson	65	$4550
Jackson	40	$3000
Callahan	50	$3500

On the basis of the information in the table, which salesman would be most likely to have the largest sales if he worked full-time?

- Wilson
- Jackson
- Callahan
- I don't know.

Ages 13, 17

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